Unique Quadrilateral and Formula

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Abstract:
In Geometry we learn about many different types of shapes and try to derive formula to generalize it. In the real life most of the objects we see around are of shape of special quadrilateral. In this paper researcher has derived a special formula for quadrilateral which is given as follows. formula $(n)^2+ (n+1)^2+(n(n+1))^2= (n(n+1)+1)^2$ *. Total fourteen observation concluded by the researcher and the fact finding results discussed in detail with the formula. Present research is also gives the proof of Pythagoras Theorem which is also provide the base for the present research. Research regarding the quadrilateral formula will open new branch of quadrilateral in geometry and will be use for the further research. From the above invented formula of Dr. Suresh is one of the formulas that can be useful to find the different study of mathematics. Description of the new formula will be useful to develop possible many riders, formula, hypotheses and true results in the field of mathematics. Brahamgupta's formula and Hero's formula for the finding Area will be constructed on the basis of cyclic quadrilateral and provide the scope of many unsolved area/problems/puzzles of the mathematics.*

Keywords: *Unsolved Area, Formula, Quadrilateral*

1. Introduction:

We may wonder why mathematics should we study. We are also eager why we learn Geometry. In our daily life we look at many shapes, and we know that all these shapes are related with geometry. Geometry is special branch of mathematics, which focus on the presentation, formation, implementation and analyzing the different logic and always tries to give them the best proof by paper-pencil and real situation of geographical is also possible in a large system and we use best without wasting unnecessary time. In Geometry we learn about many different types of shapes and try to derive formula to generalize it. We are familiar with figure of point, line, segment, angle, triangle, square, rectangle, circle etc. like as in geometry. In this paper researcher has focus on the formula of quadrilateral.

We may also wonder why we should study about quadrilateral or parallelograms. Let we look and try to understand the shape and we will find many objects which are in a shape of quadrilateral like the walls, floor, book, page, table, blackboard, duster, classroom, window… etc. Although most of the objects we see around are of shape of special quadrilateral.

In quadrilateral ABCD, AB, BC, CD and DA are four sides. A, B, C and D are Vertices angle A, angle B, angle C and angle d are the four angles formed at the vertices. Joining the opposite vertices A to C and B to D are the two diagonals of the quadrilateral ABCD. There is some formula also developed by Dr. Suresh Parmar, so in this case it is also necessary to know about Arithmetic Progression, which is given as follows.

2. Arithmetic Progression

A sequence is an arrangement of numbers in a definite order to some rule. An arithmetic progression is a sequence in which term increase or decrease regularly by the same constant. This constant is called the common difference of the progression (series). In other words, we can say that list of numbers in which the first term is given and each term is obtained by adding a fixed number to the preceding term.

In this paper researcher has derived a special formula for quadrilateral which is given as follows.

3. Unique Quadrilateral and Formula

$$(n)^2 + (n+1)^2 + (n\,(n+1))^2 = (n\,(n+1)+1)^2$$

Comparing the sides of quadrilateral:

A = (n)
B = (n+1)
C = (n (n+1))
D = (n (n+1) +1)

It is noted that in this paper quadrilateral is used in this special case side A=(n), B=(n+1), C=(n(n+1)) and D= (n(n+1)+1). So reader have to keep this information in mind for the given **quadrilateral,** because all this discovered research evidences are described under his situation of quadrilateral.

4. Description of Dr. Suresh's formula for quadrilateral

Let's try to understand the above formula in word as follows.

- Suppose there are four sides of quadrilateral.
- From those four sides of quadrilateral the measure of the very smaller side is the first side of quadrilateral with measure unit n.
- Second step is the next preceding side's of quadrilateral's measure is one unit greater than the smallest size, so the measure of the second side of quadrilateral is n+1 of quadrilateral.
- Third step is the multiplication of the both smaller sides of quadrilateral, so the measure of the third side of quadrilateral is the product of the measure of two smaller sides of quadrilateral.(n(n+1))
- Now let's sum of square of these three sides of quadrilateral and let's check the measure. By checking this measure it must be a perfect-square numbe and it is the forth side of quadrilateral.(n(n+1)+1)

Above formula can be derived by the any number putting in the Dr. Suresh's Formula.

4.1 Example: 1

Suppose the smaller side of quadrilateral is n=1
Second side of quadrilateral is n+1=1+1=2
Third side of quadrilateral is n(n+1)= 1(1+1)=1(2)=2
Now, sum of square of these three sides of the quadrilateral can be written as follows.

$(n)^2+(n+1)^2+(n(n+1))^2$
$= (1)^2+(1+1)^2+(1(1+1))^2$
$= (1)^2+(2)^2+(2)^2$
$= 1 + 4 + 4$
$= 9 = 3^2 = (1(1+1)+1)^2$

4.2 Example: 2

Suppose the smaller side of quadrilateral is n=2

Second side of quadrilateral is n+1=2+1=2
Third side of quadrilateral is n(n+1)= 2(2+1)=2(3)=6
Now, sum of square of these three sides of the quadrilateral can be written as follows.
$(n)^2+(n+1)^2+(n(n+1))^2$
$= (2)^2+(2+1)^2+(2(2+1))^2$
$= (2)^2+(3)^2+(6)^2$
$= 4 + 9 + 36$
$= 49 = 7^2= (2(2+1)+1)^2$

4.3 Example: 3
Suppose the smaller side of quadrilateral is n=3
Second side of quadrilateral is n+1=3+1=4
Third side of quadrilateral is n(n+1)= 3(3+1)=3(4)=12
Now, sum of square of these three sides of the quadrilateral can be written as follows.
$= (n)^2+(n+1)^2+(n(n+1))^2$
$= (3)^2+(3+1)^2+(3(3+1))^2$
$= (3)^2+(4)^2+(12)^2$
$= 9 + 16 + 144$
$= 169 = 13^2 = (3(3+1)+ 1)^2$

From the above three example it is clear that the sum of the square of the quadrilateral are found 9, 49 and 169. It is noted that the 4 is the square of the number 2 and the 49 is the square of the number 7 as well as 169 is square of 13. Now we compare the third side of the quadrilateral it is also comes to know that side are 2, 6 and 12, which are preceding and one less than the forth side of quadrilateral.

Table 1

First Side (n)	First Side $(n)^2$	Second Side $(n+1)^2$	Third Side $(n(n+1))^2$	Sum Square of Three Sides	Forth side and square $(n(n+1)+1)^2$
3	$(3)^2=9$	$(4)^2=16$	$(12)^2=144$	9+16+144=169	$(13)^2=169$
4	$(4)^2=16$	$(5)^2=25$	$(20)^2=400$	16+25+400=441	$(21)^2=441$
5	$(5)^2=25$	$(6)^2=36$	$(30)^2=900$	25+36+900=961	$(31)^2=961$
6	$(6)^2=36$	$(7)^2=49$	$(42)^2=1764$	36+49+1764=1849	$(43)^2=1849$
7	$(7)^2=49$	$(8)^2=64$	$(56)^2=3136$	49+64+3136=3249	$(57)^2=3249$
N	$(n)^2$	$(n+1)^2$	$(n(n+1))^2$	$(n)^2+(n+1)^2+(n(n+1))^2$	$(n(n+1)+1)^2$

5. Observation for the formula

Some of the important conclusions are also derived from the above formula. To understand the formula let's understand the Example: 1 as illustration. Some of the observation and its conclusion are given as under.

5.1 Observation 1

Unique Quadrilateral and Formula

$$(n)^2+(n+1)^2+(n(n+1))^2= (n(n+1)+1)^2$$

5.2 Observation 2

In all the side of formula 'n' is common. So, from the first side (the smallest quadrilateral) remaining three sides can be predicted correctly and they are depending in the measure of first side.

5.3 Observation 3

First side =1, second side =n+1=(1+1)=2. So it is evident that the first two smaller sides of quadrilateral are in sequence (1 and 2 are in sequence). It is also noted that the difference between the two smaller sides (2-1=1) of quadrilateral is one (1) number and the second side is proceeding of first side.

5.4 Observation 4

Third side =n(n+1)=2, Forth side =n(n+1)+1=3. So it is evident that the two greater sides of quadrilateral are in sequence (2 and 3 are in sequence). It is also noted that the difference between the two greater sides (3-2=1) of quadrilateral is one (1) number and the second side is proceeding of first side.

5.5 Observation 5

Sum of the first (smallest side) side of quadrilateral and forth (greatest side) is equal to sum of second side and third side.

First side + Forth side [1+3=4] …………… A

Second side + Third side [2+2=4] ……….. B

From the above equation A and B it can be generalized that Sum of the first side of quadrilateral and forth is equal to sum of second side and third side.

5.6 Observation 6

Third side is the product of the first side and second side [n X (n+1) = n (n+1)].

[1 X 2= 2]

Table. 2

First Side (n)	First Side (n)	Second Side(n+1)	Third Side (n(n+1))	Forth side (n(n+1)+1)
1	1	2	2	3
2	2	3	6	7
3	3	4	12	13
4	4	5	20	21
5	5	6	30	31
6	6	7	42	43
7	7	8	56	57
N	(n)	(n+1)	(n(n+1))	$(n(n+1)+1)^2$

5.7 Observation 7

From the above table no 2; it is evident that the sum of first side and the forth side is the perfect square number.

For example: 1+3=4, 3+13=16, 4+21=25, 5+31=36, 6+43=49…

5.8 Observation 8

From the above table no 2; it is evident that the sum of second side and the Third side is the perfect square number.

For example: 2+2=4, 4+12=16, 5+20=25, 6+30=36, 7+42=49…

5.9 Observation 9

[(n (n+1) X (n+1)] – [(n(n+1)+1) X (n)] = n^2 (perfect square number).

From the table no 2 it is also noted that difference between product of third side and second side and product of forth side and first side is equal to perfect square number.
4 x12= 48; 3 X 13= 39; [48-39=9=3^2]
5 x20= 100; 4 X 21= 84; [100-84=16=4^2]
6 x30= 180; 5 X 31= 155; [180-155=25=5^2]

5.10 Observation 10
[(n(n+1)+1) X (n+1)] – [n(n+1) X (n)] = n^2 (perfect square number).
From the table no 2 it is also noted that difference between product of forth side and second side and product of third side and first side is equal to perfect square number.
3 x2= 6; 2 X 1= 2; [6-2=4=2^2]
7 x3= 21; 6 X 2= 12; [21-12=9=3^2]
13 x4= 52; 12 X 3= 36; [52-36=16=4^2]
21 x5= 105; 20 X 4= 80; [105-80=25=5^2]

5.11 Observation 11
From the table no 2 it is also noted that difference between
[(n(n+1)+1) +(n+1)] – [n + n(n+1)] =2
So the difference between the sum of forth side +second side and first side + third side is equal to 2 (two).
3+2=5; 1+2=3; [5-3=2]
7+3=10; 2+6=8; [10-8]=2

5.12 Observation 12
We all know about Pythagoras Theorem:

5.12.1 Pythagoras Theorem
The square on the hypotenuse of a right-angled is equal to the sum of the squares on other two sides.
The triangle ABC is right-angled at C and the sides are enclosing the right-angled are the component of the vector which is the hypotenuse. If BC=a; CA=b and AC=c then
$AB^2 = BC^2 + CA^2$
$c^2 = a^2 + b^2$
In this formula integration of Pythagoras theorem develops new idea for thinking, which is covered and explained as under. Suppose there is a right angle between the first side and second side. According to Pythagoras theorem we can also draw a hypotenuse on these two points. Now let’s we draw a third side of quadrilateral according to Dr. Suresh Parmar’s Formula; the forth side is one greater than the third side.

5.13 Observation 13
When there is a right angle between the first side and second side. There is also right angle between that hypotenuse and the third side and joining the point of quadrilateral; according to Formula: “Sum of square two smaller sides is equal to difference between two greater side of unique quadrilateral. It can be understand by the following Table no 3.

Table 3

Sr. No.	Sum Of Square Two Smaller side	Difference between Square of two greater side
1	1+4=5	9-4=5
2	4+9=13	49-36=13
3	9+16=25	169-144=25
4	16+25=41	441-440=41
5	25+36=61	961-900=61

From the above table no 3 it is evidence that Sum of square two smaller side is equal to difference between two greater sides of quadrilateral. From the calculation data are 5,13,25,41,61 are decreasing in a special order.

Dr. Suresh Parmar has invented special formula for this kind of decreasing number and which are same for the above rule. Formula for Decreasing first number of series: 2n(n+1)+1.
By Checking the above formula by putting the value of 'n' in formula is given as under.
When n=1; 2(1) (1+1) +1= 2(2) +1= 4+1=5
n=2; 2(2) (2+1) +1= 4(3) +1= 12+1=13
n=3; 2(3) (3+1) +1= 6(4) +1= 24+1=25
n=4; 2(4) (4+1) +1= 8(5) +1= 40+1=41
n=5; 2(5) (5+1) +1= 10(6) +1= 16+1=61

5.14 Observation 14

In this paper, formula to find circumference of the quadrilateral is five as follows. Generally there is sum of measure of four side and here in this quadrilateral sides are constructed by the given formula.

Circumference of quadrilateral = (n) + (n+1) + (n (n+1)) + (n (n+1)+1)
Circumference of quadrilateral = 2n (n+2) +2
= $2n^2+4n+2$
= $2(n^2+2n+1)$
= $2(n+1)^2$

In word it can be explained as Circumference of quadrilateral is double of square of second side or it can be also said that Circumference of quadrilateral is product of two and square of second side.
Checking the above formula by putting the value of 'n' in formula is given as under.
When n=1; $2(n+1)^2=2(1+1)^{2=}2(2)^2=2(4)=8$ [1+2+2+3 =8]
When n=2; $2(n+1)^2=2(2+1)^{2=}2(3)^2=2(9)=18$ [2+3+6+7 =18]
When n=3; $2(n+1)^2=2(3+1)^{2=}2(4)^2=2(16)=32$ [3+4+12+13 =32]
When n=4; $2(n+1)^2=2(4+1)^{2=}2(5)^2=2(25)=50$ [4+5+20+21 =50]
When n=5; $2(n+1)^2=2(5+1)^{2=}2(6)^2=2(36)=72$ [5+6+30+31 =72]
When n=6; $2(n+1)^2=2(6+1)^{2=}2(7)^2=2(49)=98$ [6+7+42+43 =98]

Hence we can say that Circumference of quadrilateral;
$(n) + (n+1) + (n (n+1)) + (n (n+1) +1) = 2(n+1)^2$

6. Generalization of Formula for Quadrilateral Developed by Dr. Suresh

- $(n)^2+(n+1)^2+(n(n+1))^2= (n(n+1)+1)^2$

- In all the side of formula 'n' is common. So, from the first side (the smallest quadrilateral) remaining three sides can be predicted correctly and they are depending in the measure of first side.
- First two smaller sides of quadrilateral are in sequence. It is also noted that the difference between the two smaller sides of quadrilateral is one (1) number and the second side is proceeding of first side.
- Two greater sides of quadrilateral are in sequence. It is also noted that the difference between the two greater sides of quadrilateral is one (1) number and the second side is proceeding of first side.
- Third side is the product of the first side and second side.
- Sum of the first (smallest side) side of quadrilateral and forth (greatest side) is equal to sum of second side and third side.
- Sum of first side and the forth side is the perfect square number.
- Difference between product of third side and second side and product of forth side and first side is equal to perfect square number.
- Difference between product of forth side and second side and product of third side and first side is equal to perfect square number.
- $[(n(n+1)+1) +(n+1)] – [n + n(n+1)] =2$
- Difference between the sum of forth side +second side and first side + third side is equal to 2 (two).
- A right angle between the first side and second side. According to Pythagoras theorem we can also draw a hypotenuse on these two points. Now let's we draw a third side of quadrilateral according to Formula; the forth side is one greater than the third side.
- When there is a right angle between the first side and second side. There is also right angle between that hypotenuse and the third side and joining the point of quadrilateral; according to Formula: "Sum of square two smaller sides is equal to difference between two greater side of Dr. Suresh quadrilateral.
- Sum of square two smaller sides is equal to difference between two greater sides of quadrilateral. From the calculation data are 5, 13,25,41,61 are decreasing in a special order. Formula for Decreasing first number of Suresh series: $2n(n+2)+2$
- Circumference of quadrilateral is double of square of second side or it can be also said that Circumference of quadrilateral is product of two and square of second side.
- $(n) +(n+1) +(n(n+1)) + (n(n+1)+1) = 2(n+1)^2$

7. Conclusion

From the above invented formula by Dr. Suresh is one of the formulas that can be useful to find the different study of mathematics. Description of the new formula will be useful to develop possible many riders, formula, hypotheses and true results in the field of mathematics. Brahamgupta's formula or Hero's formula for the finding Area will be constructed on the basis of cyclic quadrilateral and provide the scope of many unsolved area/problems/puzzles of the mathematics. Interested readers of the mathematics kindly give your suggestion, viewpoint, problems related to mathematics on the following mail or e-mail address desire4sureshparmar@gmail.com.

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